LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2015**

MT 1503 - ANALYTICAL GEOMETRY OF 2D,TRIG. & MATRICES

Date : 25/09/2015 Dept. No. Max. : 100 Marks

Time : 01:00-04:00

PART – A **(10 X 2 = 20)**

Answer ALL questions

1. Write the expansion for $\cos n\theta$.

2. Write $\cos\theta$ and $\sin\theta$ in ascending powers of θ

3. Show that $\sin(ix) = i\sinh x$ and $\cos(ix) = \cosh x$

4. Find the value of log(4 + 3i).

5. Prove that the matrix $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ is orthogonal.

6. Find the rank of the matrix $A = \begin{pmatrix} 3 & -1 & 2 \\ -6 & 2 & -4 \\ -3 & 1 & -2 \end{pmatrix}$.

7. Find the polar of (3, 4) with respect to $y^2 = 4ax$

8. Define the conjugate diameters of the ellipse.

9. Write the standard form of the equation to the Rectangular hyperbola and its asymptotes.

10. Give the equation to the straight line and the conic in polar form.

PART – B **(5 X8 = 40)**

Answer any FIVE questions

11. Expand $\cos 8\theta$ in terms of $\sin\theta$

12. Evaluate $\lim_{x\to 0}\frac{\tan 2x - 2\tan x}{x^3}$

13. If $\tan\frac{x}{2} = \tanh\frac{y}{2}$ prove that $\sinh y = \tan x$ and $y = \log\tan\left(\frac{\pi}{2}+\frac{x}{2}\right)$

14. Separate into real and imaginary parts $\tan^{-1}(x+iy)$.

15. Find the characteristic equation of $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ and hence find its inverse.

16. Find the locus of the poles of chords of a parabola subtending a right angle at the vertex.

17. Prove that the acute angle between two conjugate diameters of an ellipse is minimum when they are equal.

18. If e and e_1 are the eccentricities of a hyperbola and its conjugate, Show that $\frac{1}{e^2}+\frac{1}{e_1^2}=1$.

PART– C **(2 X 20 = 40)**

Answer Any TWO Questions

19. a. Determine a, b, c such that $\lim_{\theta\to 0}\frac{\theta(a+b\cos\theta)-c\sin\theta}{\theta^5}=1$

b. Expand $\sin^3\theta\cos^4\theta$ in terms of multiples of θ.

20. a. If $\sin(A+iB)=x+iy$, prove that $\frac{x^2}{\sin^2 A}-\frac{y^2}{\cos^2 A}=1$ and $\frac{x^2}{\cosh^2 B}+\frac{y^2}{\sinh^2 B}=1$

b. If $\cos(x+iy)=r(\cos\alpha+i\sin\alpha)$, Show that $y=\frac{1}{2}\log\left[\frac{\sin(x-\alpha)}{\sin(x+\alpha)}\right]$

21. Diagonalize the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$.

22. a. P and Q are extremities of two conjugate diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and S is the focus. Prove that $PQ^2 - (SP - SQ)^2 = 2b^2$.

b. Trace curve $\frac{12}{r}=4+\sqrt{3}\cos\theta+3\sin\theta$.

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